Prep-Year Mid-Term Exam Model A March 23, 2015

Total Mark: 20 Mathematics 1-B (Algebra) Time: 30 Minutes

[1]Complete the following statements:

(a)A square matrix A is called symmetric if……………………………………………..

(b)A square matrix A has inverse if………………………………………………………

(c)The eigenvalues of a symmetric matrix of real numbers are………………………………

and the eigenvectors are…………………………………………………………………...

[2]If $A = \begin{bmatrix} 1 & 3 \\ 1 & -1 \end{bmatrix}$. Find: (a)Find the eigenvalues and eigenvectors of A

(b)Find A^n (c)Find the eigenvalues of A^{10}, 10^A, A^{-1}

[3] If $A = \begin{bmatrix} 1 & -1 & 1 \\ 0 & 2 & 1 \\ 1 & -1 & 3 \end{bmatrix}$. Show that $A.A\grave{}$ is symmetric matrix.

Model Answer

[1](a)A square matrix A is called symmetric if $\mathbf{A^t = A}$.

(b)A square matrix A has inverse if $|\mathbf{A}| \neq \mathbf{0}$.

(c)The eigenvalues of a symmetric matrix of real numbers are **real numbers and the eigenvectors are linear independent and orthogonal.**

-- ***(3 Marks)***

[2](a) $|A-\lambda I| = \begin{vmatrix} 1-\lambda & 3 \\ 1 & -1-\lambda \end{vmatrix} = (1-\lambda)(-1-\lambda) - 3 = \lambda^2 - 4 = 0$

Then, the eigenvalues are: $\lambda_1 = 2, \quad \lambda_2 = -2.$

From the equation, $\begin{bmatrix} 1-\lambda & 3 \\ 1 & -1-\lambda \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = 0$

For : $\lambda_1 = 2$, $\begin{bmatrix} -1 & 3 \\ 1 & -3 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = 0$

Then – x + 3y = 0, x – 3y = 0

Then x = 3y = any number except 0

Put y = 1, we get x = 3 and the

corresponding eigenvector is: $X_1 = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$

For : $\lambda_2 = -2$, $\begin{bmatrix} 3 & 3 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = 0$

Then 3x + 3y = 0, x + y = 0

Then y = – x = any number except 0

Put x = 1, we get y = – 1 and the

corresponding eigenvector is: $X_2 = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$

-- ***(3 Marks)***

(b)Since $T = \begin{bmatrix} 3 & 1 \\ 1 & -1 \end{bmatrix}$. Then $T^{-1} = -\frac{1}{4}\begin{bmatrix} -1 & -1 \\ -1 & 3 \end{bmatrix} = \frac{1}{4}\begin{bmatrix} 1 & 1 \\ 1 & -3 \end{bmatrix}$

Then $A^n = T.\begin{bmatrix} 2^n & 0 \\ 0 & (-2)^n \end{bmatrix}.T^{-1} = \begin{bmatrix} 3 & 1 \\ 1 & -1 \end{bmatrix}.\begin{bmatrix} 2^n & 0 \\ 0 & (-2)^n \end{bmatrix}.\frac{1}{4}\begin{bmatrix} 1 & 1 \\ 1 & -3 \end{bmatrix}$

$= \frac{1}{4}\begin{bmatrix} 3(2)^n + (-2)^n & 3(2)^n - 3(-2)^n \\ (2)^n - (-2)^n & (2)^n + 3(-2)^n \end{bmatrix}$

-- ***(4 Marks)***

(c) The eigenvalues of $f(A) = A^{10}$ are $f(2) = 2^{10}$, $f(-2) = (-2)^{10}$.

The eigenvalues of $f(A) = (10)^A$ are $f(2) = (10)^2$, $f(-2) = (10)^{-2}$.

The eigenvalues of $f(A) = A^{-1}$ are $f(2) = 2^{-1}$, $f(-2) = (-2)^{-1}$.

-- ***(3 Marks)***

[3] $A.A` = \begin{bmatrix} 1 & -1 & 1 \\ 0 & 2 & 1 \\ 1 & -1 & 3 \end{bmatrix}\begin{bmatrix} 1 & 0 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 3 \end{bmatrix} = \begin{bmatrix} 3 & -1 & 5 \\ -1 & 5 & 1 \\ 5 & 1 & 11 \end{bmatrix}$. We see that A.A` is symmetric

-- ***(2 Marks)***

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Prep-Year **Mid-Term Exam** **Model AA** **March 23, 2015**

Total Mark: 20 **Mathematics 1-B (Algebra)** **Time: 30 Minutes**

[1]Complete the following statements:

(a)A square matrix A is called non-singular if……………………………………………..

(b)The determinant of a matrix exists if…………………………………………………….

(c)If $\lambda_1, \lambda_2, \ldots, \lambda_n$ are all eigenvalues of a matrix A, then |A| is …………………………..

[2]If $A = \begin{bmatrix} 0 & 1 \\ 3 & 2 \end{bmatrix}$. Find: (a)Find the eigenvalues and eigenvectors of A

(b)Find A^n (c)Find the eigenvalues of $\frac{10}{A^2+I}$, $\sqrt{A^2 + 2I}$

[3] Find A^2 where $A = \begin{bmatrix} 2 & -2 & -4 \\ -1 & 3 & 4 \\ 1 & -2 & -3 \end{bmatrix}$

Model Answer

[1](a)A square matrix A is called non-singular if $|\mathbf{A}| \neq \mathbf{0}$.

(b)The determinant of a matrix exists if **it is a square matrix**.

(c)If $\lambda_1, \lambda_2, \dots, \lambda_n$ are all eigenvalues of a matrix A, then |A| is $\boldsymbol{\lambda_1 . \lambda_2 . \dots, \lambda_n}$.

-- ***(3 Marks)***

[2](a) $|A - \lambda I| = \begin{vmatrix} 0-\lambda & 1 \\ 3 & 2-\lambda \end{vmatrix} = (-\lambda)(2-\lambda) - 3 = \lambda^2 - 2\lambda - 3 = 0$

Then, the eigenvalues are: $\lambda_1 = 3, \quad \lambda_2 = -1.$

From the equation, $\begin{bmatrix} -\lambda & 1 \\ 3 & 2-\lambda \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = 0$

For : $\lambda_1 = 3$, $\begin{bmatrix} -3 & 1 \\ 3 & -1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = 0$

Then – 3x + y = 0, 3x – y = 0

Then y = 3x = any number except 0

Put x = 1, we get y = 3 and the

corresponding eigenvector is: $X_1 = \begin{bmatrix} 1 \\ 3 \end{bmatrix}$

For : $\lambda_2 = -1$, $\begin{bmatrix} 1 & 1 \\ 3 & 3 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = 0$

Then x + y = 0, 3x + 3y = 0

Then y = – x = any number except 0

Put x = 1, we get y = – 1 and the

corresponding eigenvector is: $X_2 = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$

-- ***(3 Marks)***

(b)Since $T = \begin{bmatrix} 1 & 1 \\ 3 & -1 \end{bmatrix}$. Then $T^{-1} = -\frac{1}{4}\begin{bmatrix} -1 & -1 \\ -3 & 1 \end{bmatrix} = \frac{1}{4}\begin{bmatrix} 1 & 1 \\ 3 & -1 \end{bmatrix}$

Then $A^n = T.\begin{bmatrix} 3^n & 0 \\ 0 & (-1)^n \end{bmatrix}.T^{-1} = \begin{bmatrix} 1 & 1 \\ 3 & -1 \end{bmatrix}.\begin{bmatrix} 3^n & 0 \\ 0 & (-1)^n \end{bmatrix}.\frac{1}{4}\begin{bmatrix} 1 & 1 \\ 3 & -1 \end{bmatrix}$

$$= \frac{1}{4}\begin{bmatrix} (3)^n + 3(-1)^n & (3)^n - (-1)^n \\ (3)^{n+1} - 3(-1)^n & (3)^{n+1} + (-1)^n \end{bmatrix}$$

-- ***(4 Marks)***

(c) The eigenvalues of $f(A) = \frac{10}{A^2+I}$ are $f(3) = \frac{10}{9+1} = 1$, $f(-1) = \frac{10}{1+1} = 5.$

The eigenvalues of $f(A) = \sqrt{A^2 + 2I}$ are $f(3) = \sqrt{9+2} = \sqrt{11}$, $f(-1) = \sqrt{1+2} = \sqrt{3}$

-- ***(3 Marks)***

[3] $A^2 = A.A = \begin{bmatrix} 2 & -2 & -4 \\ -1 & 3 & 4 \\ 1 & -2 & -3 \end{bmatrix}\begin{bmatrix} 2 & -2 & -4 \\ -1 & 3 & 4 \\ 1 & -2 & -3 \end{bmatrix} = \begin{bmatrix} 2 & -2 & -4 \\ -1 & 3 & 4 \\ 1 & -2 & -3 \end{bmatrix}$

-- ***(2 Marks)***

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